Stable Bloch oscillations and Landau-Zener tunneling in a non-Hermitian $\mathcal{PT}$-symmetric flat-band lattice

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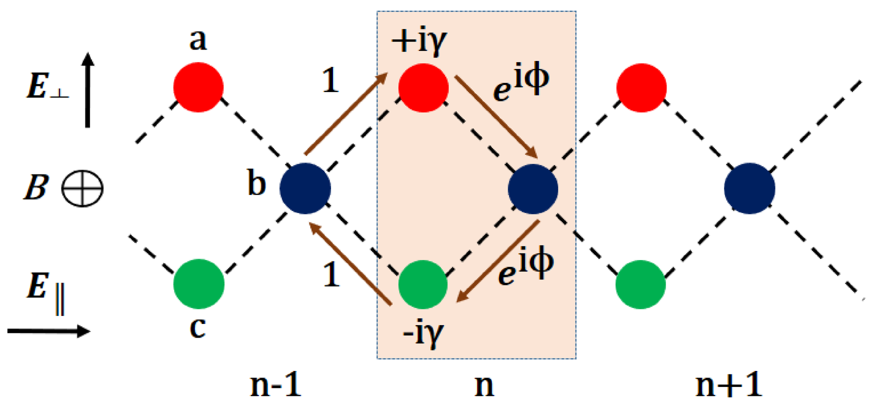
Abstract

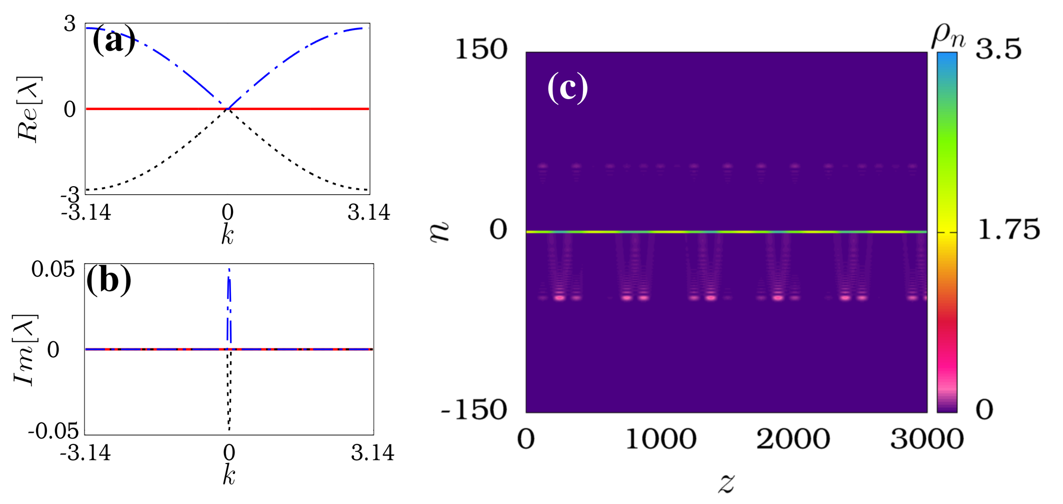
- To study the Bloch oscillations (BOs) and Landau-Zener(LZ) tunneling in a non-Hermitian system in the presence of external fields
- We investigate a non-Hermitian $\mathcal{PT}$-symmetric diamond chain network and its transport dynamics in two different situations,
  (i) Flat band case,
  (ii) Non-Flat band case,
- The system does not support completely real eigenspectra in any of the parametric regions in both cases which leads to the broken $\mathcal{PT}$ phase.
- Our results reveal that super Bloch oscillations can be observed in the completely broken $\mathcal{PT}$-phase of the system when exposed to the synthetic electric field.

Model



Schematic diagram of non-Hermitian diamond chain lattice made up of an array of waveguides.

Asymmetric BOs



(a) and (b) Real and imaginary parts of the band structure for a $\mathcal{PT}$-symmetric system with phase $\phi = \pi$ and in the absence of synthetic electric field components ($E_\parallel = 0$, $E_\perp = 0$). Gain-loss parameter is chosen as $\gamma = 0.05$. The solid red line represents the non-dispersive (flat) band. Dashed black and dash-dotted blue curve represents the complex dispersive nature of the bands. (c) Asymmetric Bloch oscillations corresponding to the flat band case for the CLS initial excitation given in Eq. (4) with $E_\parallel = 0.05$.

System equations

- The dynamics of the evolving electric field amplitude at the n^{th} unitcell of an array of non-Hermitian lattice is given by the following equations,

$$
\begin{aligned}
i\dot{a_n} &= (E_\parallel n + E_\perp)a_n + i\gamma a_n - e^{-i\phi}b_n - b_{n-1},\\
i\dot{b_n} &= E_\parallel(n+\frac{1}{2})b_n - e^{i\phi}a_n - e^{-i\phi}c_n - c_{n+1} - a_{n+1},\\
i\dot{c_n} &= (E_\parallel n - E_\perp)c_n - i\gamma c_n - e^{i\phi}b_n - b_{n-1}.
\end{aligned} \tag{1}
$$

where,
a_n, b_n, c_n → complex field amplitudes in the waveguides,
z → propagation distance in dimensionless unit,
γ → gain and loss parameter introduced through a proper choice of the complex refractive index profiles,
$E_\parallel$, $E_\perp$ → components of the synthetic electric field along the longitudinal and transverse directions to the lattice plane,
ϕ → magentic flux introduced through the direction dependent phase factor between the waveguides results in the synthetic magnetic field B.

- The considered system as given in Eq. (1) is $\mathcal{PT}$-symmetric in the absence of transverse electric field where the system is invariant under the combined operation of parity and time-reversal symmetries defined by $a_n \to -c_n$, $b_n \to -b_n$, $c_n \to -a_n$, $i \to -i$ and $z \to -z$.

Flat band and compact localized modes

The dispersion relation of the considered system given in (1) in the presence of synthetic magnetic field with phase $\phi = \pi$ and in the presence of both transverse electric field and magnetic field are shown below,

$$\phi = \pi: \quad \lambda_1 = 0, \quad \lambda_{2,3} = \mp\sqrt{-\gamma^2 + 4(1-\cos k)}. \tag{2}$$

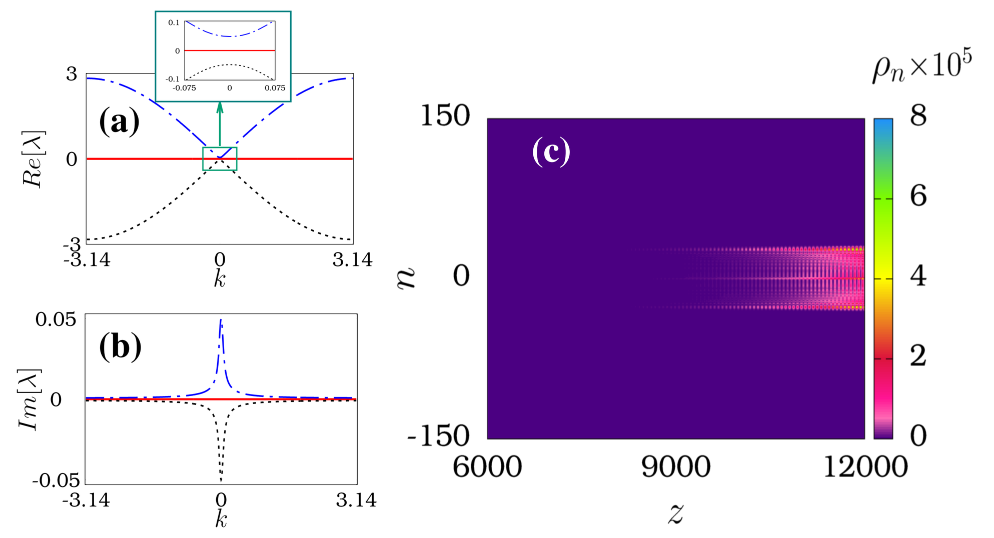
$$\phi = \pi, E_\perp \neq 0: \lambda_1 = 0, \quad \lambda_{2,3} = \mp\sqrt{E_\perp^2 + 2i\gamma E_\perp - \gamma^2 + 4(1-\cos k)}. \tag{3}$$

The compact localized modes (CLS) which include non-zero amplitudes at a finite number of sites and vanishing amplitudes at all other sites related to Eq. (2) and (3) takes the form,

$$
\begin{aligned}
A_n &= -C_n = (-1)^n A_0 \delta_{s,n}, \quad s = 0 \ \ and \ \ 1,\\
B_n &= -i\gamma A_0 \delta_{0,n}.
\end{aligned} \tag{4}
$$

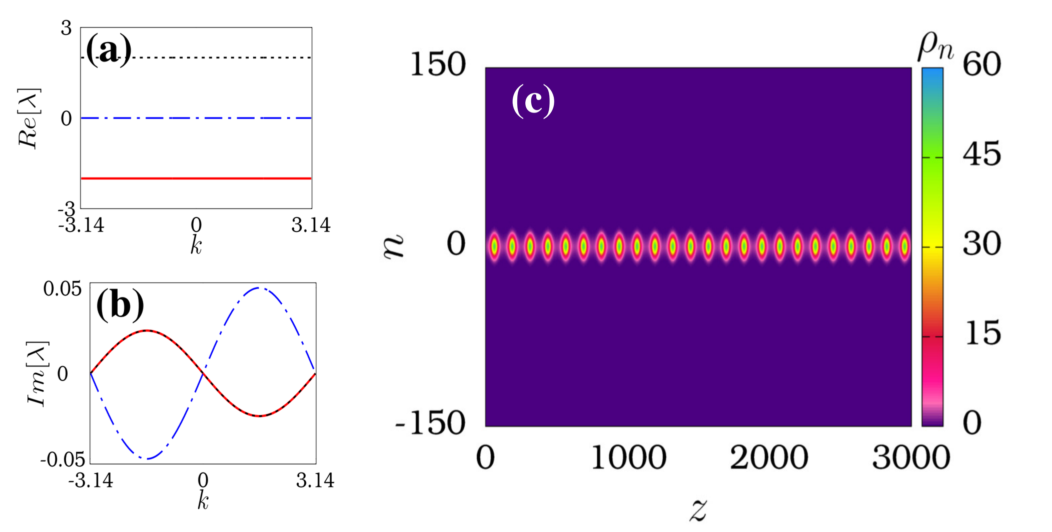
$$
\begin{aligned}
A_n &= -C_n = (-1)^n A_0 \delta_{s,n}, \quad s = 0 \ \ and \ \ 1\\
B_n &= (-E_\perp - i\gamma) A_0 \delta_{0,n}.
\end{aligned} \tag{5}
$$

Amplified BOs-LZ tunneling



(a) Real and (b) Imaginary parts of the band structures in the presence of $E_\perp = 0.05$ shows the emergence of bandgaps. (c) Amplified Bloch oscillations due to Landau-Zener tunneling for CLS excitation given in (5) with $E_\parallel = 0.1$, $\phi = \pi$ and $\gamma = 0.05$.

Super BOs



(a) Real and (b) Imaginary parts of the band structures for $\phi = \frac{\pi}{2}$, $\gamma = 0.05$, $E_\parallel = 0.0$ and $E_\perp = 0.0$. (c) shows the existence of super Bloch oscillations for Gaussian excitation in the presence of the longitudinal electric fields $E_\parallel = 0.05$.

Conclusions

- Our model supports flat band only in particular situation and we have identified the asymmetric nature of the Bloch oscillations observed in the flat band case
- When the flat and dispersive bands are found to be isolated, we observed amplifying Bloch oscillations through Landau-Zener tunneling in the presence of electric fields
- These results can be applicable in optical communications to enhance the optical signals during propagation.
- Considering the non-flat band case, landau-zener tunneling makes super Bloch oscillations possible in the broken phase of the system and it may be useful in the applications of optical amplification or in achieving localized transport of a high-intense beam.

References

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Acknowledgments

J.R.P. thanks the Department of Science and Technology, Government of India, for providing an INSPIRE Fellowship No. DST/INSPIRE Fellowship/2017/IF170539. The work of V.K.C. forms part of the research projects sponsored by SERB-DST-MATRICS Grant No. MTR/2018/000676 and CSIR Project Grant No. 03(1444)/18/EMR-II. J.R.P and V.K.C also wishes to thank SASTRA Deemed University for extending infrastructure and support to carry out this work. M.L. wishes to thank the Department of Science and Technology for the award of a SERB Distinguished Fellowship under Grant No. SB/DF/04/2017.